A computational method for normalization of Trapezoidal Fuzzy Numbers

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ABSTRACT

In multi criteria decision making, if the judgments are expressed as fuzzy numbers, the normalization process is necessary in obtaining the rank of alternatives. Extent analysis method on Fuzzy Analytical Hierarchy Process is a popular decision making method introduced by D.Y. Chang(1996). This method was applied on various applications to obtain the rank of alternatives. The normalization method is to normalize Triangular Fuzzy Numbers, it has some flaws according to Wang and Elhag (2006). This paper presents the modified extent analysis method with accurate normalization procedure and an extent analysis method was introduced to find the weights of alternatives when the judgments are provided as Trapezoidal Fuzzy Numbers.

Key words: *Analytical Hierarchy Process, Fuzzy Analytical Hierarchy Process, Extent Analysis Method, Trapezoidal Fuzzy Numbers.*

1. Introduction

Analytical Hierarchy Process (AHP) is one of the power ful decision making technique applied on many practical decision making problems which is developed by T. L. Saaty [13]. AHP uses relative scales to rank the alternatives. Judgments can be expressed as paired comparisons to compare the criteria or alternatives in decision making problems. Evangelos Triantaphyllou [7] applied this method on some practical issues of industrial engineering. To choose from three alternatives to upgrade the computer system in terms of four decision criteria: hard ware maintainability, user friendly, hardware expandability, financing availability the author [7] applied AHP. Buckly [2] employed fuzzy numbers to rank alternatives in decision making problem and discussed how to pool experts' opinions.

Trapezoidal fuzzy numbers are used to collect the opinions of experts. Buckly [3] extended his previous model using Geometric Mean Method on reciprocal matrix formed by experts opinions as pair-wise comparison. Reshma Radakrishnanet.al [11] employed Fuzzy AHP to select best school children. Liang and Wang [10] employed Trapezoidal Fuzzy Numbers in selection of a site by developing an algorithm. Carlsson and Fuller [4] categorized fuzzy multi criteria methods into three groups and introduced a new method. Fatemah and Lewis [8] fuzzy multi criteria decision making methods are more powerful for decision making since they express all kinds of ambiguity. Dubois and Prade [6] introduced methods for the comparison of fuzzy numbers. They applied these methods in multi criteria decision making methods. Zhu et. Al [16] modified the formula proposed by Chang.D.Y[5].

By applying on practical problem in petroleum industry they modified the comparison of Triangular Fuzzy Numbers proposed by Chang.Tang[12] applied Fuzzy AHP on problem of budget allocation in aerospace engineering. Kousalya et.al applied AHP and Fuzzy AHP for selection of the student for awarding all round excellence award and did comparison study. They examined the results by weights comparison obtained by both the methods.

2. Methodology

Laarhoven and Pedrycz [9] worked Triangular Fuzzy Numbers in Fuzzy AHP. Chang D. Y.[5] proposed Extent Analysis method on Fuzzy AHP. This method was applied on various practical applications. In this method expert's opinions were represented as Triangular Fuzzy Numbers. Trapezoidal Fuzzy Numbers were more general form with comparing to Triangular Fuzzy Numbers a methodology was developed to represent the experts' opinions as Trapezoidal Fuzzy Numbers.

According to Abhinav Bansal [1] arithmetic operators on Trapezoidal Fuzzy Numbers are given as follows:

Let $\widetilde{M}_1 = (e_1, f_1, g_1, h_1)$ and $\widetilde{M}_2 = (e_2, f_2, g_2, h_2)$be two trapezoidal fuzzy numbers then

(i) $\widetilde{M}_1 + \widetilde{M}_2 = (e_1, f_1, g_1, h_1) + (e_2, f_2, g_2, h_2) = (e_1 + e_2, f_1 + f_2, g_1 + g_2, h_1 + h_2)$

(ii) $\widetilde{M}_1 \otimes \widetilde{M}_2 = (e_1, f_1, g_1, h_1) \otimes (e_2, f_2, g_2, h_2) = (e_1 e_2, f_1 f_2, g_1 g_2, h_1 h_2)$

(iii) $\widetilde{M}_1^{-1} = \left(\frac{1}{h_1}, \frac{1}{g_1}, \frac{1}{f_1}, \frac{1}{e_1}\right)$

According to Wang.Y.M. & Elhag T.M.S [15], if an weight interval vector $W = (w_1, w_2, \dots., w_n)$ with $w_i = [w_i^L, w_i^U], 0 \le w_i^L \le w_i^U$ for $i = 1,2, \dots., n$ satisfies

$$\sum_{i=1}^{n} w_i^L + \max_j \left(w_j^U - w_j^L\right) \le 1, \sum_{i=1}^{n} w_i^U - \max_j \left(w_j^U - w_j^L\right) \ge 1$$

then, it is normalized otherwise, it is not normalized.

2.1 Normalization method:

For the interval weight vector $W = (w_1, w_2, \dots., w_n)$ where $w_i = [w_i^L, w_i^U], 0 \le w_i^L \le w_i^U$ for $i = 1,2, \dots., n$. Consider the following model

$Min/_{Max}\, \widehat{w}_i = \frac{z_i}{\sum_{j=1}^{n} z_j}$ such that $w_j^L \le z_j \le w_j^U, j = 1,2, \dots\dots, n.$

The model generates a weight interval$[\widehat{w}_i^L, \widehat{w}_i^U]$ for each$\widehat{w}_i$, where $\widehat{w}_i^L = Min\, \widehat{w}_i$and $\widehat{w}_i^U = Max\, \widehat{w}_i$.

The first order partial derivatives of $\widehat{w}_i$ with respect to each $z_j, j = 1,2, \dots., n$ are given below

$$\frac{\partial \widehat{w}_i}{\partial z_i} = \frac{\sum_{j=1}^{n} z_j - z_i}{\left(\sum_{j=1}^{n} z_j\right)^2} = \frac{\sum_{j \ne i}^{n} z_j}{\left(\sum_{j=1}^{n} z_j\right)^2} > 0, \frac{\partial \widehat{w}_i}{\partial z_j} = -\frac{z_i}{\left(\sum_{j=1}^{n} z_j\right)^2} < 0, j = 1,2, \dots., n; j \ne i$$

It is clear that, $\widehat{w}_i$ is increases with z_i, and decreases with $z_j, j = 1,2, \dots., n;\ j \ne i$. It's maximum and minimum can be written as follows.

$$\widehat{w}_i^L = \widehat{w}_i^{min} = \frac{w_i^L}{w_i^L + \sum_{j\neq i}^{n} w_j^U}, i = 1,2,\ldots,n \ and\ \widehat{w}_i^U = \widehat{w}_i^{max} = \frac{w_i^U}{w_i^U + \sum_{j\neq i}^{n} w_j^L}, i = 1,2,\ldots,n$$

Dubois and Prade[6] are firstly developed the above two equations.

2.2 Extended Extent Analysis Method:
We obtain the following matrix for each level of hierarchy

$$\tilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} (1,1,1,1) & (e_{12}, f_{12}, g_{12}, h_{12}) & \cdots & (e_{1n}, f_{1n}, g_{1n}, h_{1n}) \\ (e_{21}, f_{21}, g_{21}, h_{21}) & (1,1,1,1) & \cdots & (e_{2n}, f_{2n}, g_{2n}, h_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (e_{n1}, f_{n1}, g_{n1}, h_{n1}) & (e_{n2}, f_{n2}, g_{n2}, h_{n2}) & \cdots & (1,1,1,1) \end{bmatrix}$$

Where $\tilde{a}_{ij} = (e_{ij}, f_{ij}, g_{ij}, h_{ij})$ and $\tilde{a}_{ij}^{\ -1} = \left(\frac{1}{h_{ij}}, \frac{1}{g_{ij}}, \frac{1}{f_{ij}}, \frac{1}{e_{ij}}\right), i = 1,2,\ldots,n; j = 1,2,\ldots,n.$
This represents the judgments for the alternatives and criteria.

First Step: Fuzzy synthetic extent value with respect to i[th] object is defined as

$$\tilde{S}_i = \frac{RS_i}{\sum_{j=1}^{n} RS_j} = \left(\frac{\sum_{j=1}^{n} e_{ij}}{\sum_{j=1}^{n} e_{ij} + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} h_{ij}}, \frac{\sum_{j=1}^{n} f_{ij}}{\sum_{j=1}^{n} f_{ij} + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} g_{kj}}, \frac{\sum_{j=1}^{n} g_{ij}}{\sum_{j=1}^{n} g_{ij} + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} f_{kj}}, \frac{\sum_{j=1}^{n} h_{ij}}{\sum_{j=1}^{n} h_{ij} + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} e_{kj}}\right) \quad \text{------- (1)}$$

To obtain RS_i, perform the fuzzy addition operation of n extent analysis values for a particular matrix such that:

$$RS_i = \sum_{j=1}^{n} \tilde{a}_{ij} = \left(\sum_{j=1}^{n} e_{ij}, \sum_{j=1}^{n} f_{ij}, \sum_{j=1}^{n} g_{ij}, \sum_{j=1}^{n} h_{ij}\right)$$

Second Step: As $\widetilde{M}_1 = (e_1, f_1, g_1, h_1)$ and $\widetilde{M}_2 = (e_2, f_2, g_2, h_2)$ are the two Trapezoidal fuzzy numbers as shown in Figure 1, the possible degree of $\widetilde{M}_2 = (e_2, f_2, g_2, h_2) \geq \widetilde{M}_1 = (e_1, f_1, g_1, h_1)$ is defined as

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = \sup_{y \geq x}\left(min\left(\mu_{\widetilde{M}_1}(x), \mu_{\widetilde{M}_2}(y)\right)\right) \quad \text{----------- (2)}$$

The above is equivalent to

$$V(\widetilde{M}_2 \geq \widetilde{M}_1) = hgt(\widetilde{M}_2 \cap \widetilde{M}_1) = \mu_{\widetilde{M}_2}(d) = \begin{cases} 1, & if\ g_2 \geq f_1 \\ 0, & if\ e_1 > h_2 \\ \dfrac{e_1 - h_2}{e_1 - h_2 + g_2 - f_1}, & Otherwise \end{cases}$$

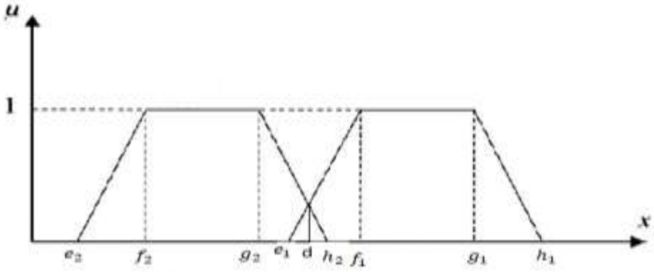

***Fig 1**: Intersection between $\widetilde{M}_2$ and $\widetilde{M}_1$*

Third Step: A possible degree for convex fuzzy numbers is defined by

$$V(M \geq M_1, M_2, M_3, \ldots, M_k) = V\lfloor (M \geq M_1) \ and \ (M \geq M_2) \ and \ldots (M \geq M_k) \rfloor$$
$$= Min\, V(M \geq M_i), i = 1,2, \ldots, k \qquad ....(3)$$

Suppose $d'(A_i) = Min\, V(S_i \geq S_k), k = 1,2, \ldots, n; k \neq i$

Weight vector is $W' = \left(d'(A_1), d'(A_2), d'(A_3), \ldots, d'(A_n)\right)^T$ ....(4)

Fourth Step: The normalized vectors via normalization are

$W = \left(d(A_1), d(A_2), d(A_3), \ldots, d(A_n)\right)^T$------- (5)

Where $d(A_1) = \frac{d'(A_1)}{\sum_{i=1}^{n} d'(A_i)}, d(A_2) = \frac{d'(A_2)}{\sum_{i=1}^{n} d'(A_i)}, \ldots, d(A_n) = \frac{d'(A_n)}{\sum_{i=1}^{n} d'(A_i)}$

Here W is a non-fuzzy number.

3. CONCLUSION and FUTURE SCOPE

The methodology can be applied for the problems on multi criteria decision making where the opinions are as trapezoidal fuzzy numbers. The normalization method explained in first step of procedure is the method developed by considering the flaws explained by Wang Y.M, Luo Y, Hua Z [14].

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